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Exclusive f_2 leptoproduction via the odderon exchange

M.G. Ryskin

Petersburg Nuclear Physics Institute, Gatchina, St.Petersburg 188350, Russia

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Abstract. Cross sections of high energy semielastic $\gamma^* p \to f_2 p$ leptoproduction of $f_2(1270)$ or $\eta(548)$ mesons as well as the $\gamma p \to f_2 + M_D^2$ photoproduction at large t are estimated in the framework of perturbative QCD using the hypothesis of parton-hadron duality.

1 Introduction

Due to a more complicated colour structure of the $SU(3)_c$ group QCD predicts the existence of a high energy amplitude with all vacuum quantum numbers in the t-channel except of the C-parity. The amplitude with $C = -1$ and intercept α^{odd} close to 1 was called "odderon" [1]. In the Born approximation the amplitude is given by the three gluon exchange, where any pair of gluons forms the symmetric colour octet 8_s. This contribution is pure real and proportional to c.m. energy square, $T^{odd} \propto s$. In other words it corresponds to $\alpha^{odd} = 1$ and does not vanish for high energy. The leading logarithmic corrections to the Born amplitude were discussed in [2–4]. Unfortunately, there are still no final results. Only the limits [5]

$$\alpha^{odd} - 1 \ < \ \frac{3}{2}(\alpha^{Pom} - 1)$$

are known (here $\alpha^{Pom} = 1 \cdot \frac{12\alpha_s}{\pi}\ln 2$ is the intercept of the BFKL pomeron [6] leading logarithmic amplitude).

The odderon exchange should reveal itself in the real part of the nucleon-nucleon amplitude. However the simplest estimate [7] shows that due to the additional power of the QCD coupling α_s and some numerical factors, at least in the Born approximation, an odderon contributes at most 0.02 to the value of the ratio Re/Im for the pp-forward amplitude. Thus it is hard to search for the odderon in pp- (or $p\bar{p}$)-elastic scattering.

It was argued in [8] that the measurement of exclusive electroproduction of mesons with positive C-parity ($C = +1$) at HERA might test the presence of the QCD odderon. The cross section of η_c photo- and electroproduction was calculated in [9].

In the present paper we will discuss the production of f_2 and $\eta(548)$ mesons (which contain only light quarks). As the wave function of the $f_2(1270)$ meson is not known well enough the hypothesis of parton-hadron duality will be used. Strictly speaking, we will calculate the cross section of light quark $q\bar{q}$-pair exclusive electroproduction, selecting the state with fixed mass $M_{q\bar{q}} = M_{f_2} \pm \Delta M$ and the $J^{PC} = 2^{++}$ ($I^G = 0^+$) quantum numbers. After the hadronization such a state will form mainly the f_2-meson; there is not enough phase space to produce (with large probability) a more complicated multiparticle state with such a low mass and $J^P = 2^+$. In any case our result may be considered as upper limit for the cross section of $\gamma^* + p \to f_2 + p$.

Both electroproduction by longitudinally and transversely polarized photons will be discussed. As it is known for the $\gamma^* \to \rho$ production the transverse cross section is suppressed by the factor M^2/Q^2 in comparison with the longitudinal one. In contrast we will show that the $\gamma^* \to f_2$ production amplitude falls down faster with Q^2 for the longitudinal photon and $\sigma^L/\sigma^T \propto M^2/Q^2$ for $\gamma \to f_2$.

In Sect. 3 we will discuss the large t "inelastic" process with the dissociation of the target proton into some diffracted state M_D^2. As in this case there is no proton form-factor the $\gamma \to f_2$ photoproduction cross section decreases slowly with t ($d\sigma/dt \sim 1/t^3$). Taking into account the possibility of a large $O(\alpha_s)$ corrections (due to the $O(C_F \frac{\alpha_s}{\pi}\pi^2)$ K-factor and the leading $(\alpha_s \ln 1/x)^n$ terms, which correspond to the positive odderon intercept [1]) one may hope to expect relatively large cross section $\sigma(\gamma p \to f_2 + M_D^2) \simeq 0.5$ nb (for $|t| > 10$ GeV^2) which can be observed at HERA.

2 Born amplitude

In the Born approximation the quark-quark scattering via the odderon exchange is given by the three gluon amplitude (Fig. 1)

$$T(Q,s) = \frac{10\alpha_s^3}{9^2\pi} s \int \frac{d^2q_1 d^2q_2 d^2q_3 \delta^{(2)}(q - q_1 - q_2 - q_3)}{q_1^2 \ q_2^2 \ q_3^2}, \tag{1}$$

where q_i is the momentum transferred along the gluon $i = 1,2,3$ (at high energy $s \gg q_i^2$ the 4-momentum square $q_i^2 \simeq q_{it}^2$ — the transverse component square) and q is the

[1] We hope that $\Delta^{odd} > 0$

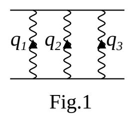

Fig. 1. Born diagram for the odderon exchange in the quark-quark scattering

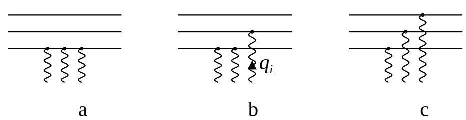

Fig. 2. Coupling of the odderon (3 gluons) to the nucleon (3 quarks) wave function

momentum transferred along the odderon as a whole. The numerical factor $10/9^2\pi$ takes into account the symmetry of the gluons (1/3!), the sum over the gluon's colour indices ($\sum_{ijk}|d_{ikj}|^2 = 40/3$), the averaging over the colours of the incoming quarks (1/9) and the factor $1/2\pi$ due to the Feynman loop integration.

The odderon coupling to the proton vertex may by written in the form [7,9]:

$$F(q_i) = 3\left\{G(q^2) - \sum_{i=1}^{3} G(q^2 - 3q_i q + 3q_i^2) + 2G\left(\sum_{i=1}^{3}\frac{3q_i^2}{2} - \frac{q^2}{2}\right)\right\}. \quad (2)$$

Here the first term corresponds to the graphs, where all three gluons couple to the same quark line (Fig. 2a). The second term reflects the contribution from diagrams where only two gluons are connected to the same quark (see Fig. 2b), wheres the last term stems from diagrams of the type shown in Fig. 2c, where any gluon couples to another valence quark.

Expression (2) is evident in the case of the non-relativistic quark wave function of a proton (with the oscillatoric potential and a Gaussian form-factor $G(Q^2) = \exp(R^2Q^2)$, $Q^2 < 0$). Following [9] we will use expression (2) also in the case of the dipole form-factor $G(Q^2) = 1/(1 + |Q^2|/0.71\ \mathrm{GeV}^2)^2$ in order to estimate how the result depends on the form of the nucleon wave function.

An important property of the vertex (2) is the fact that due to the colourless nature of the nucleon the function $F(q_i)$ vanishes for any momentum $q_i \to 0$. This property provides the infrared convergence of the integrals dq_i^2/q_i^2 in the amplitude of the type of (1).

In principle we whould have to discuss the exact form of the $\gamma^* \to f_2$ vertex. However, as we don't know the wave function of f_2-meson well we will use the hypothesis of parton-hadron duality and consider the $\gamma^* \to q\bar{q}$ process as the basic production mechanism.

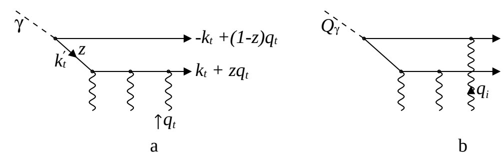

Fig. 3. Coupling of the odderon to the quark-antiquark (meson) state

2.1 Longitudinal $\gamma^* \to q\bar{q}(J^P = 2^+)$ amplitude

As in the case of 'elastic' ρ electroproduction [10] it is convenient to use the formalism of [11,12] and to calculate first the matrix element of the $\gamma \to q\bar{q}$ transition putting both the quarks (with helicities λ and $\lambda' = \pm 1$) on mass shell. For the longitudinally polarized initial photon the matrix element reads

$$\Psi_{\lambda,\lambda'}(k'_t, z) = \frac{e_q\sqrt{z(1-z)}}{\bar{Q}^2 + k_t'^2}\bar{u}_\lambda \gamma \cdot \epsilon v_{\lambda'} = 2Q\,\frac{e_q z(1-z)}{\bar{Q}^2 + k_t'^2}\,\delta_{\lambda,-\lambda'}, \quad (3)$$

where $Q = \sqrt{-Q_\lambda^2}$, $\bar{Q}^2 = z(1-z)Q_\gamma^2$, e_q is the electric charge of the light quark, ϵ is the photon polarization vector, k'_t and z are the transverse momentum and photon momentum fraction carried by the quark (see Fig. 3). The quark-gluon vertex conserves the helicity of high energy quark and the momentum fraction z is not changed also. Therefore only the transverse momentum of the quark k'_t in (3) is different from the final quark momentum $k_t + zq_t$. In order to simplify the projection onto the $J^P = 2^+$ state we use the momentum k_t in the rest frame of the $q\bar{q}$-system (the z axis is directed along the target proton momentum) and put the quark mass $m_q = 0$. So the value of $k_t = (M/2)\sin\theta$ (where $M = M_{q\bar{q}}$ is the mass of the $q\bar{q}$-pair and θ is the quark polar angle).

In this notation the contribution from the Feynman diagram of the type shown in Fig. 3a reads

$$2Qe_q z(1-z)\delta_{\lambda,-\lambda'}\ \frac{1}{\bar{Q}^2 + (k_t - (1-z)q_t)^2}\ . \quad (4)$$

Summing up all graphs with any permutations of the three t-channel gluons leads to

$$S^L = \frac{1}{\bar{Q}^2 + (k_t - (1-z)q_t)^2} - \frac{1}{\bar{Q}^2 + (k_t + zq_t)^2} + \sum_{i=1}^{3}\left[\frac{1}{\bar{Q}^2 + (k_t + zq_t - q_{it})^2} - \frac{1}{\bar{Q}^2 + (k_t - (1-z)q_t + q_{it})^2}\right]. \quad (5)$$

Here the second term corresponds to the same Fig. 3a type diagram, but where all the three t-channel gluons couple to the antiquark $\bar{q}$ instead of the quark q, while the other terms reflect the permutations of a single gluon ($i =$

1, 2, 3). At large photon virtuality Q^2 when $|\bar{Q}^2| \gg |k_t'^2|$ the leading contributions ($\sim 1/\bar{Q}^2$) in (5) cancel and the first non-zero contribution of the expansion $1/(\bar{Q}^2+k'^2) \simeq 1/\bar{Q}^2 - k'^2/\bar{Q}^4 + k'^4/\bar{Q}^6$ is of the order $1/\bar{Q}^6$. Indeed:

$$
\begin{aligned}
S^L \simeq \frac{1}{\bar{Q}^4} \Bigg\{ & (k+zq)_t^2 - \sum_i (k+zq-q_i)_t^2 \\
& + \sum_i (k+(z-1)q+q_i)_t^2 - (k+(z-1)q)_t^2 \Bigg\} \\
& + \frac{1}{\bar{Q}^6} \Big[(k+zq)_t^4 - (k+(z-1)q)_t^4 \\
& + \sum_i (k+(z-1)q+q_i)_t^4 - \sum_i (k+zq-q_i)_t^4 \Big] \\
= \frac{1}{\bar{Q}^6} \Bigg[& ((k+zq)q_t) \left(4q^2 - 4\sum_i q_i^2 \right) \\
& + 8 \sum_i ((k+zq)q_{it})(q_i^2 - q_t q_i) \\
& + 2q^4 + \sum_i \left(4q_i^2(qq_i) - 4(q_i q)^2 - 2q_i^2 q^2 \right) \Bigg], \qquad (6)
\end{aligned}
$$

where the expression in the curly brackets gives zero after taking into account the fact that $\sum_i q_i = q$. Using the identity

$$
\begin{aligned}
& 2\sum_i (qq_i)(q_i^2 - (qq_i)) + q^4 - q^2 \sum_i q_i^2 = \qquad (7) \\
& 2\Big[(qq_1)(q_2q_3) + (qq_2)(q_1q_3) + (qq_3)(q_1q_2) \Big] =: M(q,q_i)q^2
\end{aligned}
$$

the vertex factor S^L may be written as

$$
S^L \simeq \frac{4}{Q^6} \left[(kq_t) + \left(\frac{1}{2} + z \right) q^2 \right] M(q, q_i) \; . \qquad (8)
$$

Now it is easy to see that the main logarithmic contribution to the odderon exchange amplitude

$$
\begin{aligned}
& {}^L T^{odd} = \qquad (9) \\
& \frac{10\alpha_s^3}{9^2\pi} s \int \frac{d^2q_1 d^2q_2}{q_1^2 q_2^2 q_3^2} F(q_i) 2Qz(1-z) S^L(k_t, z, \bar{Q}, q_i)
\end{aligned}
$$

comes from the region of large $q_i \gg q, 1/R$ (but $q_i \ll \bar{Q}$). Function $F(q_i)$ is defined in (2) and for $q_i \gg q, 1/R$ only the first term in (2), $F(q_i) \simeq 3G(q^2)$ corresponding to Fig. 2a is essential. So for $1/R$, $q \ll q_i \ll \bar{Q}$ the integral (9) takes the form

$$
\begin{aligned}
I = 2 & \int \frac{d^2q_1 d^2q_2 d^2q_3' \delta^{(2)}(q - q_1 - q_2 - q_3')}{q_1^2 q_2^2 q_3'^2} \\
& \times \Big[(qq_1)(q_2q_3') + (qq_2)(q_1q_3') + (qq_3')(q_1q_2) \Big] \\
\approx 2q^2 & \int \frac{d^2q_1 d^2q_2 d^2q_3 \delta^{(2)}(q_1 + q_2 + q_3)}{q_1^2 q_2^2 q_3^2} \\
& \times \left[(q_2q_3) - q_2^2 + \frac{2(q_2q_3)^2}{q_3^2} \right] + O(q^4) \; ,
\end{aligned}
$$

where we put $q_3' = q + q_3$ and keep only terms of order q^2, averaging over the direction of the vector $\mathbf{q}_t$ in the azimuthal plane.

With the help of Feynman parameter x it is now easy to get:

$$
\begin{aligned}
I &= 2 \int_0^1 \frac{dx \; d^2q_2 d^2q_3}{[(1-x)q_2^2 + xq_1^2]^2} \frac{q^2[...]}{q_3^2} \\
&= \int \frac{dx \; q^2 \; d^2(q_2 + xq_3)}{[q_2^2 + 2x(q_2q_3) + xq_3^2]^2} 2\frac{d^2q_3}{q_3^2} \\
&\quad \times \Big[-q_2'^2 + 2xq_2'q_3 - x^2q_3^2 + q_2'q_3 - xq_3^2 \\
&\qquad + \frac{2(q_2'q_3)^2}{q_3^2} + \frac{2x^2q_3^4}{q_3^2} - \frac{4(q_2'q_3)xq_3^2}{q_3^2} \Big] \\
&= 2\pi \int \frac{q^2 dx(x^2 - x)}{(x - x^2)q_3^2} \; d^2q_3 \\
&= -2\pi^2 q^2 \ln \frac{\bar{Q}^2}{q_c^2}
\end{aligned}
$$

(here $q_2' = q_2 + xq_3$ and the lower limit $q_c \simeq \max\{q/3, 1/3R\}$). Thus

$$
\begin{aligned}
{}^L T^{\,odd} \simeq \frac{80\pi\alpha_s^3}{27} s \frac{(kq_t) + (1/2 + z)q^2}{\bar{Q}^6} G(q^2) \\
\times \ln \frac{\bar{Q}^2}{q_c^2} \cdot 2Qz(1-z). \qquad (10)
\end{aligned}
$$

To obtain the total cross section of $\gamma^* \to q\bar{q}$ dissociation we have to integrate over the upper quark loop

$$
\begin{aligned}
& \frac{d\sigma^L}{dtdM^2} \\
& = \sum_{\lambda,\lambda'} \frac{4\pi\alpha e_q^2 N_c}{16^2\pi^4 s^2} \int dz d^2k_t \delta \left(M^2 - \frac{k_t^2 + m_q^2}{z(1-z)} \right) \\
& \times |T^{odd}|^2 \delta_{\lambda,-\lambda'} \qquad (11)
\end{aligned}
$$

($\alpha = 1/137$ is the electromagnetic coupling, $N_c = 3$ and we put the light quark mass $m_q = 0$).

Now we perform the integration over k_t using the δ-function ($k_t^2 = z(1-z)M^2$) and write the integral over dz in terms of the polar angle: $z = \frac{1}{2}(1+\cos\theta)$. The last step is to separate the $q\bar{q}$-state with definite $J^P = 2^+$

$$
\frac{d\sigma^L}{dtdM^2} \; = \; \sum_{\lambda,\lambda'} \frac{\alpha e_q^2 N_c}{(16\pi)^2} \sum_J |{}^L C_m^J|^2 \delta_{\lambda,-\lambda'} \qquad (12)
$$

with the projection

$$
{}^L C_m^J \; = \int_{-1}^{1} d_{mm'}^J(\theta) \frac{{}^L T^{odd}}{s} \sqrt{z(1-z)} \; d\cos\theta \sqrt{2J+1}. \qquad (13)
$$

An extra factor $\sqrt{z(1-z)}$ was generated in (13) by the $\delta[M^2 - k_t^2/(z(1-z))]$ function after the dk_t^2 integration.

The lower indices of the $d^J_{mm'}$-function reflect the projection of the total spin J onto the initial photon axis (m) and onto the quark axis ($m' = (\lambda - \lambda')/2 = \pm 1$ due to helicity conservation of the $\gamma \to q\bar{q}$-vertex). At zero angle (corresponding to $q_t = 0$) the projection $m = 0$ for σ^L and $m = 1$ for σ^T. However, for non-zero q_t, the term $(k_t q)$ in (10) corresponds to $m = 1$ (for σ^L) due to the orbital momentum of the $q\bar{q}$-pair motion (the factor k_t reflects the p-wave nature of the amplitude). Thus both configuration with $m = 1$ and $m = 0$ are produced by the longitudinal photon via the odderon exchange.

The integration over $dz = \frac{1}{2} d\cos\theta$ in (13) has the logarithmic form $dz/z(1-z)$. Indeed, in the denominator of the amplitude (10) we have $\bar{Q}^6 = Q^6 z^3 (1-z)^3$ and one additional factor $\sin\theta = 2\sqrt{z(1-z)}$ comes either from the quark momentum $k_t = (M/2)\sin\theta$ or from the function $d^2_{01}(\theta) = \sqrt{3/2}\sin\theta\cos\theta$. Therefore in the leading logarithmic approximation (LLA) the coefficients C^J_m take the form:

$$ {}^L C^2_0 = \frac{q^2}{Q^5} G(q^2) \frac{80\pi\sqrt{15}}{27\sqrt{2}} \alpha_s^3 \, 4 \ln^2 \frac{Q^2}{4q_c^2} \quad (14) $$

$$ {}^L C^2_1 = \frac{M|q|}{Q^5} G(q^2) \frac{40\pi\sqrt{5}\,\alpha_s^3}{27} \; 4 \ln^2 \frac{Q^2}{4q_c^2} \; . \quad (15) $$

The main contribution to the value of C^J_m comes from the logarithmically large interval of z where $\bar{Q}^2 = z(1-z)Q^2 = \sin^2\theta\,(Q^2/4)$ varies from $q_i^2 \sim (q_t/3)^2$ up to $\bar{Q}^2 = Q^2/4$.

The $q^2 = t$ behaviour of the Born cross section $d\sigma^2/dt$ is evident from (12,14,15). For the numerical estimates we use $M = M_{f_2} = 1270$ MeV, $\alpha_s = 0.4$, $R^2 = 2.75$ GeV^{-2}. The projection onto the isospin $I^G = 0^+$ state gives $\sum_{q=u,d} e_q^2 \otimes 0^+ = 1/9$. Note that in accordence with the parton-hadron duality the cross section (12)–(15) was integrated over the interval $\Delta M^2 = 1$ GeV^2.

Of course the cross section vanishes at $t = 0$. It has a maximum at $|t| \simeq 0.3$ GeV^2. But even after the $t-$integration its value ($\simeq 1.5$ pb at $Q^2 = 10$ GeV^2) is probably too small to be observed at HERA.

2.2 Transverse $\gamma^* \to q\bar{q}$ ($J^P = 2^+$) amplitude

For the transversely polarized photon the matrix element of the $\gamma \to q\bar{q}$ transition takes the form [11,12]

$$ \begin{aligned} \Psi_{\lambda,\lambda'}(k'_t, z) &= \frac{e_q\sqrt{z(1-z)}}{\bar{Q}^2 + k'^2_t} \bar{u}_\lambda \gamma \epsilon_\pm v_{\lambda'} \\ &= \frac{e_q \delta_{\lambda,-\lambda'}}{\bar{Q}^2 + k'^2_t} (\epsilon_\pm k'_t)[(1-2z)\lambda \mp 1], \end{aligned} \quad (16) $$

where $\epsilon_\pm = \frac{1}{\sqrt{2}}(0,0,1,\pm i)$ is the photon polarization vector. After the summation over the permutations of the three t-channel gluons we obtain (in analogy with (5)):

$$ \begin{aligned} S^T &= \frac{(\epsilon(k + zq - q)_t)}{\bar{Q}^2 + (k + zq - q)^2_t} - \frac{(\epsilon(k + zq))}{\bar{Q}^2 + (k + zq)^2_t} \quad (17) \\ &+ \sum_{i=1}^{3} \left[\frac{(\epsilon(k + zq - q_i)_t)}{\bar{Q}^2 + (k + zq - q_i)^2_t} - \frac{(\epsilon(k + zq - q + q_i)_t)}{\bar{Q}^2 + (k + zq - q + q_i)^2_t} \right] \\ &\approx \frac{1}{\bar{Q}^4} \left[2 \sum_i (\epsilon q_i)\left(q_i^2 - (qq_i)\right) + (\epsilon q)\left(q^2 - \sum_i q_i^2\right) \right]. \end{aligned} $$

As in the previous case (Sect. 2.1) the leading log contribution comes from the region of $q_i \gg q$, and finally

$$ {}^T T^{\,odd} = \frac{s}{\bar{Q}^4} \, (\epsilon q_t) G(q^2) \, \frac{40\alpha_s^3}{27} \pi \Phi(z) \ln \frac{\bar{Q}^2}{q_c^2} \; , \quad (18) $$

where the function $\Phi(z) = 1 - z$ for $\lambda = 1$ or $\Phi(z) = z$ for $\lambda = -1$ (for the case of $\epsilon = \epsilon_-$). Only the state with the projection $m = 0$ is produced by the transverse photon at large Q^2 and

$$ \begin{aligned} {}^T C^J_0 &= \int_{-1}^{1} d^J_{0\lambda}(\theta)\, {}^T T^{odd} \sqrt{2J+1} \sqrt{z(1-z)} d\cos\theta \\ &\simeq \frac{q_t}{Q^4} G(q^2) \sqrt{\frac{15}{2}} \frac{80\pi}{27} \alpha_s^3 \ln^2 \frac{Q^2}{4q_c^2}. \end{aligned} \quad (19) $$

The total cross section $\sigma^T \simeq 1.7$ pb at $Q^2 = 10$ GeV^2 is close to the longitudinal one σ^L. However at larger Q^2 the longitudinal component $\sigma^L \propto 1/Q^{10}$ falls down faster than $\sigma^T \propto 1/Q^8$.

The straightforward numerical computations show that the accuracy of the leading logarithmic expressions for the integral I (see (9,10)) is better than 20%. Unfortunately another terms of the nucleon-odderon vertex $F(q_i)$ (see (2)) are important at not very high values of $Q^2 \sim 10 - 100$ GeV^2. The whole amplitude ${}^L T^{odd}$ becomes smaller at $|t| < 0.2$ GeV^2 then changes the sign and has a maximum (of its absolute value) at $|t| \sim 1$ GeV^2 (at $|t| \sim 0.7$ GeV^2 for ${}^T T^{odd}$). The height of the maximum is typically close ($\pm 20\%$) to the value of maximum of the LLA expressions (14, 15).

More serious problem is the contribution coming from the infrared region of $q_{it} < 200 - 300$ MeV. In spite of the fact that all the integrals are convergent and all the singularities are integrable this contribution in not negligible. In all the computations the infrared $1/q_i^2$ singular behaviour of the gluon propagators was softened by adding the small mass ($m_g = 200$ MeV), i.e. we put $1/(q_i^2 - m_g^2)$ instead of $1/q_i^2$ in the denominator of (9). If, for example, one replaces $m_g = 200$ MeV by $m_g = 300$ MeV the cross section decreases by about factor of two. Without any infrared cutoff ($m_g = 0$) the cross section increases about 4 times in comparison with the case of $m_g = 200$ MeV ($\alpha_s = 0.4$ is still fixed). However it is hard to believe that the perturbative QCD is valid in such a “soft” region. therefore we consider the result with $m_g = 1/1\text{Fm} = 200$ MeV as a more realistic one. Of course the predictions are uncertain by a factor of 2 - 4.

In order to see the role of the target proton wave function we compare the exponential parametrization of the proton form-factor ($G(q^2) = \exp(R^2 q^2)$, $R^2 = 2.75$

GeV^{-2} in agreement with the electromagnetic proton radius) and the dipole parametrization $G(q^2) = 1/(1 + |q^2|/(0.71\ \text{GeV}^2))^2$. Within the 10-20% accuracy the results are close to each other and reveal the same qualitative features.

2.3 η-production

To estimate the cross section of $\eta(548)$-meson electroproduction one has to convolute the amplitude with the $d^0_{00} \equiv 1$ function. This gives the contribution only in the case of transversely polarized photon where for the massive quarks the matrix element has the component $m\lambda\delta_{\lambda\lambda'}$.

$$\bar{u}_\lambda(\gamma^T\epsilon_\pm)v_{\lambda'} = \frac{1}{\sqrt{z(1-z)}}\left\{\delta_{\lambda,-\lambda'}(\epsilon_\pm k'_t)[(1-2z)\lambda \mp 1] + \lambda m\delta_{\lambda\lambda'}\right\} . \quad (20)$$

As now for any $\lambda = \lambda' = \pm 1$ the projection onto the quark axis $m' = (\lambda - \lambda')/2$ is equal to zero we have to sum up over the λ just at the amplitude level [2]. So the η-production is induced mainly by the strange ('heavy') quark and

$$\frac{d\sigma^2}{dtdM^2} = \frac{\alpha e_s^2 N_c}{(16\pi)^2}\,|\langle s\bar{s} \mid \eta\rangle|^2|C^0_{00}|^2 . \quad (21)$$

With: $e_s^2 = 1/9$ and $\langle s\bar{s} \mid \eta\rangle = -2/\sqrt{6}$ in the case of unbroken flavour $SU(3)_F$ (or $\langle s\bar{s} \mid \eta\rangle \simeq -1/\sqrt{3}$ if one takes into account the realistic mixing angles) one gets

$$C^0_{00} = mq^2\int_0^1 dz\frac{80\pi\alpha_s^3}{27}G(q^2) \times \ln\frac{\bar{Q}^2}{q_0^2}\,\frac{2\sqrt{z(1-z)}\,(z+1/2)}{Q^6z^3(1-z)^3 + O(q^2+m^2)} . \quad (22)$$

The factor 2 comes from the sum over $\lambda = \pm 1$; the fact that the element $\lambda m\delta_{\lambda\lambda'}$ changes the sign together with λ reflects the pseudoscalar nature of the η-meson; $\eta = 1/\sqrt{2}(q_\uparrow\bar{q}_\downarrow - q_\downarrow\bar{q}_\uparrow)$.

Unfortunately, the integral (22) is not infrared stable. The main contribution comes from the region of $\bar{Q}^2 = Q^2z(1-z) \sim q^2$ (i.e. $z(1-z) \sim q^2/Q^2$), where (if the momentum transfer q^2 is not too large) the perturbative QCD approach is not justified. Therefore we can present only a very crude estimate [3]

$$C^0_{00} \sim \frac{4}{3}\frac{mq^2}{Q^3}G(q^2)\,4\frac{80\pi\alpha_s^3}{27}\,\frac{1}{(q^2+m^2)^{1.5}} \quad (23)$$

leading to $\sigma^\eta \propto 1/Q^6$ ($\sigma^\eta \sim 0.6\,\text{pb}$ for $Q^2 = 10\ \text{GeV}^2$).

[2] In Sect. 2 where $\lambda = \pm 1$ corresponds to the different values of $m' = \pm 1$ there was no interference and it was sufficient to sum over λ at the level of the cross section (12)

[3] Note also that for such a small $\bar{Q}^2 \sim q_i^2 \sim q^2$ strictly speaking we cannot use the expansion (6) over $1/\bar{Q}^2$ and one has to integrate the function S^L (5) explicitely

3 Large t photoproduction

As we have already seen the main contribution to the cross section comes from not too small $q^2 = t$ ($|t| \sim 1\text{GeV}^2$). Thus let us consider another possibility to study the odderon exchange, namely, the measurement of the f_2-meson photoproduction at rather large momentum transfer $-t \equiv q^2$. Here, $Q^2_\gamma = 0$ and the large scale (needed to justify the perturbative QCD approach) is controlled by the value of $|t|$.

To avoid the suppression coming from the target proton form-factor one may consider the "inelastic" process with the dissociation ($\gamma^* + p \to f_2 + M_D^2$) of the proton into a 'small' mass system $M_D^2 \sim |t|$.

The cross section is given by the same formulae (12), (16) as in Sect. 2. However now (at $Q^2 = 0$) one cannot use the decomposition over the small parameter $q_i^2/\bar{Q}^2$ and instead of (17) we have to integrate the expression for S^T explicitely. In the limit $Q \to 0$ and $k \le M/2 \ll q = \sqrt{|t|}$ the amplitude

$${}^T T^{\,odd} \simeq \frac{10\alpha_s^3}{9^2\pi}\, s\int\frac{d^2q_1d^2q_2}{q_1^2q_2^2q_3^2}F(x')\,S^t \quad (24)$$

with

$$S^t \simeq 2\Phi(z)\left\{\frac{\epsilon q(z-1)}{(1-z)^2q^2} - \frac{\epsilon qz}{z^2q^2} + \sum_{i=1}^{3}\left[\frac{\epsilon(zq-q_i)_t}{(zq-q_i)^2} - \frac{\epsilon((z-1)q+q_i)_t}{((z-1)q+q_i)^2}\right]\right\} \quad (25)$$

Let us for a moment put $i = 1$. Then the integral over dq_2 may be done with the help of the Feynman parameter x

$$\int\frac{d^2q_2}{(\mu^2+q_2^2)((q-q_1-q_2)^2+\mu^2)} = \int_0^1\frac{dx}{[q_2^2(1-x)+(q'-q_2)^2x+\mu^2]^2} = \int_0^1\frac{dx}{\mu^2+q'^2x(1-x)} = \frac{\pi}{\sqrt{q'^2(4\mu^2+q'^2)}}\ln\left|\frac{2\mu^2+q'^2+\sqrt{q'^2(4\mu^2+q'^2)}}{2\mu^2+q'^2-\sqrt{q'^2(4\mu^2+q'^2)}}\right| , \quad (26)$$

where $q_3 = q - q_1 - q_2$, $q' = q - q_1$ and a small infrared cutoff μ^2 was included. Due to the fact that $S^t \to 0$ for any momenta $q_i \to 0$ the final expression is infrared stable (i.e. tends to the finite limit at $\mu \to 0$).

In (24) the function $F(x')$ is the probability amplitude to find an appropriate target quark. In the LLA[4]

$$F^2(x') = \sum_{f=u,d,s}(f(x',q^2) + \bar{f}(x',q^2))dx' . \quad (27)$$

[4] Note that in LLA the odderon does not couple to the gluon-parton

In order to estimate the scale of the cross section (integrated over the region where M_D^2 is not too large) we will put $\int F^2(x')dx' = 3$ below, which means that only the valence quarks are taken into account.

With a reasonable ($\sim 10\%$) accuracy the results of the numerical integration (over d^2q_1 and dz) may be approximated by the simple formula

$$C \simeq 22(1-3.3\mu/q)^4 \tag{28}$$

and the cross section of f_2 photoproduction[5] is then given by

$$\frac{d\sigma}{dt\,dM^2} \simeq \frac{\alpha_s^6}{t^3} 2\frac{\alpha e_q^2 N_c}{(16\pi)^2} 3|C|^2 \tag{29}$$

$$\simeq \frac{4.4}{|t|^3}\left(1-3.3\frac{\mu}{\sqrt{|t|}}\right)^8 \left[\frac{nb}{\mathrm{GeV}^4}\right]$$

In the region of $|t| > 10$ GeV^2 this corresponds to the cross section $\sigma(|t| > 10 \text{ GeV}^2) \simeq 15$ pb for $\Delta M^2 = 1$ GeV^2, $\mu = 200$ MeV, and $\alpha_s = 0.4$. Note that without the infrared cutoff ($\mu = 0$) the cross section increases about 7 times reaching the value $\sigma_{(\mu=0)}(|t| > 10 \text{ GeV}^2) \simeq 100$ pb. However one can not believe the perturbative QCD prediction coming from the region of so small momenta. Thus the first estimate ($\sigma(|t| > 10 \text{ GeV}^2) \simeq 15$ pb) looks more realistic.

For η-meson photoproduction due to the factor m in the matrix element (20) (instead of the momentun $k_t' \sim q$ in the case of f_2-meson) the cross section falls down faster with $|t|$ and the expected $d\sigma^\eta/dt\,dM^2 \propto m^2/t^4$.

4 Conclusion

As it was shown in Sect. 2 the Born amplitudes for the QCD odderon exchange is too small to lead to the observable f_2- (or η-) meson electroproduction cross sections at HERA. At $Q^2 = 10$ GeV^2 the expected ($\gamma^* p \to f_2 p$) and ($\gamma^* p \to \eta p$) Born cross sections are about 1 pb.

The case of photoproduction at large t looks more promissing. In Born approximation the cross section of f_2-production in the region of $|t| > 10$ GeV^2 ($Q_\gamma^2 = 0$) $\sigma(\gamma p \to f_2 + M^2) \sim 15$ pb. Moreover, there is some hope that higher order α_s corrections will increase this value at least by an order of magnitude. Indeed:

1. In analogy with the Drell–Yan process the so-called K-factor, which comes mainly from the contribution of the form $C_F \frac{\alpha_s}{\pi} \cdot \pi^2$ (in the photon-odderon vertex), most probably enlarges the cross section of about two or four times (see [10] for more detail).
2. The summation of the leading $\log 1/x$ corrections

$$\sum_n c_n \left(\alpha_s \ln\frac{1}{x}\right)^n$$

to the odderon exchange Born amplitude may shift the position (intercept) of the odderon singularity to the right in the complex momentum j-plane. The only known result at the moment is the semi-classical estimate of Korchemsky [13]. He gives $\Delta^{odd} = \alpha^{odd} - 1 \simeq 0.87\Delta^P \simeq 0.2 - 0.3$ (if one uses the latest F_2 data to estimate the value of Δ^P). Even for $\Delta^{odd} = 0.2$ this factor [6]

$$\sigma \propto (1/x)^{2\Delta^{odd}} = \left(\frac{W^2}{m_{f_2}^2 + |t|}\right)^{2\Delta^{odd}} \simeq 15$$

(for $W = 100$ GeV and $t = -10$ GeV^2) increases the cross section by more than a factor of 10. Therefore one may hope that the realistic (including the higher order α_s corrections) estimate is $\sigma(\gamma p \to f_2 + M_D^2) \simeq 0.5$nb (at $|t| > 10$ GeV^2) and such a process may be studied at the present HERA collider.

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[5] To be more specific, — the photoproduction of a $q\bar{q}$-pair in the $J^P = 2^+$, $I^G = 0^+$ state

[6] Strictly speaking $\Delta^{odd} = \alpha^{odd} - 1 \leq 0.87\Delta^P$ [13]